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Corrections for Tropospheric Range Error

EDWARD E. ALTSHULER

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Range error corrections calculated from various models of index of refraction generally require the use of numerical methods and a large-capacity high-speed computer, particularly for low elevation angles. This paper presents a simple empirical expression for range error for elevation angles above 5°. The CRPL Reference Atmosphere 1958 was the model used. A simple regression line that yields range error corrections within 1 percent of the values obtained by means of more sophisticated techniques was derived.

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MICROWAVE PHYSICS LABORATORY PROJECT 8682

AIR FORCE CAMBRIDGE RESEARCH LABORATORIES
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AIR FORCE SYSTEMS COMMAND
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Range error corrections calculated from various models of index of refraction generally require the use of numerical methods and a large-capacity high-speed computer, particularly for low elevation angles. This paper presents a simple empirical expression for range error for elevation angles above 5^{o}. The CRPL Reference Atmosphere 1958 was the model used. A simple regression line that yields range error corrections within 1 percent of the values obtained by means of more sophisticated techniques was derived.

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1. INTRODUCTION

Satellite systems like the 621B have been proposed for air traffic control and navigation. Such systems are designed to provide aircraft position and velocity data based on time delay measurements of propagation between aircraft and a network of satellites.

Since the index of refraction of the troposphere is greater than unity, propagation of radiowaves through the troposphere is slower than through freespace. Also, the variation in the index of refraction as a function of altitude produces a bending of the propagated wave, with the net result an increase in the path length.

To determine the tropospheric range error that produces the time delay, the index of refraction of the troposphere along the ray path must be known. A number of models of the index of refraction as a function of altitude have been used for calculating the tropospheric range error. The expressions obtained are rather complicated, however, particularly for low elevation angles. Numerical methods and a large-capacity high-speed computer must be resorted to for a solution, which may nevertheless be difficult to achieve in real time under operational conditions.

The atmospheric index of refraction decreases approximately exponentially with altitude, and so range error decreases with increasing elevation angle. For

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angles above 15° the range error correction can be obtained from a very simple expression. This paper presents the derivation of a simple but accurate empirical expression for range error correction for elevation angles down to 5°.

2. REFRACTIVITY MODELS

Numerous refractivity (N) profiles have been produced over the years, from samples measured directly with a refractometer or calculated from measurements of temperature, pressure, and relative humidity. A number of models have been generated from these data for the purpose of correcting for tropospheric effects on propagation. Historically, the earth's radius model formulated by Schelleng et al (1933) was first used for line-of-sight communications problems. It was shown that by assuming an earth having a radius of about 4/3 that of the actual earth, radio rays could be drawn as straight lines.

The 4/3 earth model works well for propagation paths at low altitudes, where ray paths are within about 2 km of the earth's surface, but not for those at higher altitudes. From an examination of many years of N-profile data for various climates it appears that N decreases approximately exponentially above an altitude of about 1 km and that the variation in refractivity is minimum at an altitude of 9 km, with a range of about only 8 N units and an average value of 104.8 N units (Bean and Dutton, 1966, p. 61). The Rocket Panel data (1952), the ARDC Model Atmosphere 1956 (1957) and work by Dubin (1954) show that above 9 km, refractivity decreases exponentially. These data were used in formulating the model known as the CRPL Reference Atmosphere 1958, which includes the following expressions for refractivity for three ranges of altitude:

$$N(h) = N_s + (h - h_s)\,\Delta N, \qquad h_s \le h \le h_s + 1\ \text{km},$$

where

$$\Delta N = -7.32\,\exp(0.005577)\,N_s;$$

$$N(h) = N_1 \exp[-c(h - h_s - 1)], \qquad h_{s+1} \le h \le 9\ \text{km},$$

where

$$c = \frac{1}{8 - h_s} \ln \frac{N_1}{105};$$

$$N(h) = 105 \exp[-0.1424(h - 9)], \quad h \ge 9\ \text{km},$$

where

$N(h)$ = refractivity at altitude h,

N_s = surface refractivity,
N_1 = refractivity at altitude 1 km above the surface,
h_s = height above sea level.

To simplify these expressions, the following single exponential (referred to as the CRPL Exponential Reference Atmosphere 1958) was generated:

$$N(h) = N_s \exp[-c_e(h - h_s)] ,$$

where

$$c_e = \ln \frac{N_s}{N_s + \Delta N} .$$

Although this model is in good agreement with refractivity data below 8 km, the values it gives at higher altitudes are too low. For example, at an altitude of 9 km the average refractivity is about 105, with minimum and maximum values of 100 and 108 respectively; but at this altitude the Exponential Reference Atmosphere gives a value of only 85 for N_s = 313 (average surface refractivity in the United States). This model is therefore not expected to be very useful for range error calculations.

Hopfield (1969) noted that if the refractivity as a function of height is represented by an exponential, it is not integrable in closed form, whereas if it has the form

$$N = k(h_{o_\mu} - h)^\mu , \quad h \leq h_{o_\mu} ,$$

it is integrable. She showed that representing the dry and wet terms of refractivity by quartic equations (μ = 4) gave good agreement with range error and doppler data (Hopfield, 1971).

A number of other models of refractivity have been proposed. For the most part they are variations of those mentioned above and will not be discussed.

Probably the best source of refractivity data is the CRPL Standard Atmospheric Radio Refractive Index Sample (Bean, Cahoon, and Thayer, 1960). It consists of a sample of 77 N profiles selected from thirteen radiosonde stations representative of the major geographic and climatic types of the world. Range errors were calculated for each profile as a function of elevation angle, using ray-tracing; and least-square regression lines of the form $\Delta R_e = a + bN_s$ were obtained for some angles. For elevation angles above 5^o, range errors calculated from the CRPL Reference Atmosphere 1958 were found to be in good agreement with those obtained from this regression line (Norton, 1964). The CRPL Reference

Atmosphere 1958 was therefore selected for the range error calculations. Fortunately, results based on this model were available from Millman (1970), who had generated extensive curves of range error as a function of elevation angle, surface refractivity, and height. By selecting a model consisting of stratified layers only 50 m thick, from ground level to an altitude of 30 km, and taking into accound refractive bending, he was able to compute very accurate range errors.

3. FORMULATION OF SIMPLIFIED EXPRESSION FOR RANGE ERROR

The object in the work reported here was to generate a simple expression that would provide range errors in close agreement with those computed by Millman (1970) for the CRPL Reference Atmosphere 1958 for elevation angles above 5^o. Bean and Dutton (1966, pp. 337-339), using ray-tracing techniques, had shown that range error due to bending of the propagated wave is small for elevation angles above 3^o, and had derived the following general expression for range error:

$$\Delta R_e \cong \csc\theta_o \int_o^{h_t} N\,dh + \sum_{i=1}^{\infty} (-1)^{i+1} \int_o^{h_t} N \left[\cot\theta_o \sin(\theta-\theta_o) - 2\sin^2\left(\frac{\theta-\theta_o}{2}\right)\right]^i dh. \quad (1)$$

They estimated that the summation terms contribute about 3 percent or less for $\theta_o \cong 10^o$, and about 12 percent for $\theta_o \cong 5^o$, and obtained a regression curve for

$$10^{-3}\int_o^{\infty} N(h)\,dh = 1.4588 + 0.0029611\,N_s, \quad (2)$$

which is the limiting form of the total range error based on the CRPL Standard Sample for $\theta = 90^o$.

Combining the first term on the righthand side of Eq. (1) with Eq. (2) gives:

$$\Delta R_e = \csc\theta_o \; (1.4588 + 0.0029611\,N_s)\,, \quad (3)$$

where

ΔR_e = range error in meters,
θ_o = elevation angle,
N_s = surface refractivity.

A comparison of range errors obtained from Eq. (3) with those computed by

Millman (1970) (see Figure 1) shows that the differences increase approximately exponentially with decreasing elevation angle for constant surface refractivity.

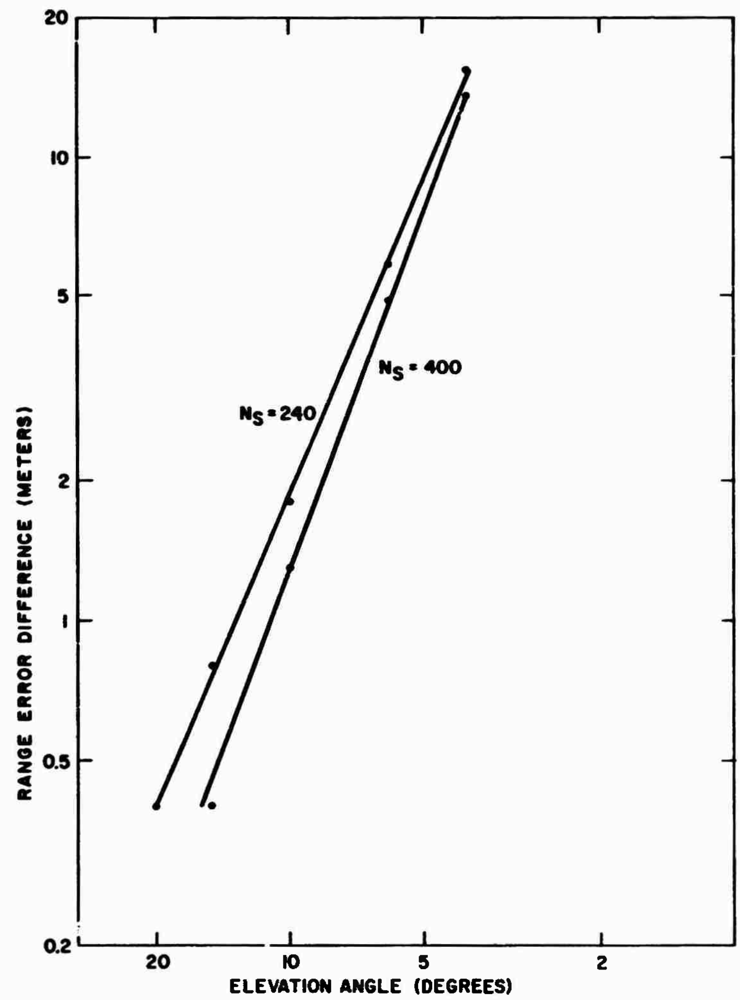

Figure 1. Differences Between Approximate and Accurate Expressions for Range Error as a Function of Angle (N_s constant)

Plotted as a function of refractivity for constant elevation angle (Figure 2) the differences are shown to be quadratic in form. Hence, adding a term of the form

$$\Delta(\theta, N_s) = \Delta\left[c_1\theta^{c_2},\ c_3(N_s - c_4)^2 + c_5\right]$$

to Eq. (3) should decrease the differences. As shown in Appendix A,

$$\Delta(\theta, N_s) = \left[0.00586\,(N_s - 360)^2 + 294\right]\theta^{-2.30}. \tag{4}$$

Therefore,

$$\Delta R_{eo} = \frac{4.79 + 0.00972\,N_s}{\sin\theta} - \left[0.00586\,(N_s - 360)^2 + 294\right]\theta^{-2.30}, \tag{5}$$

where

ΔR_{eo} = range error in feet,
θ = elevation angle in degrees,
N_s = refractivity in N units at surface.

Equation (5) yields range errors from the earth's surface through the total troposphere that are within ±0.5 ft of those computed by Millman (1970).

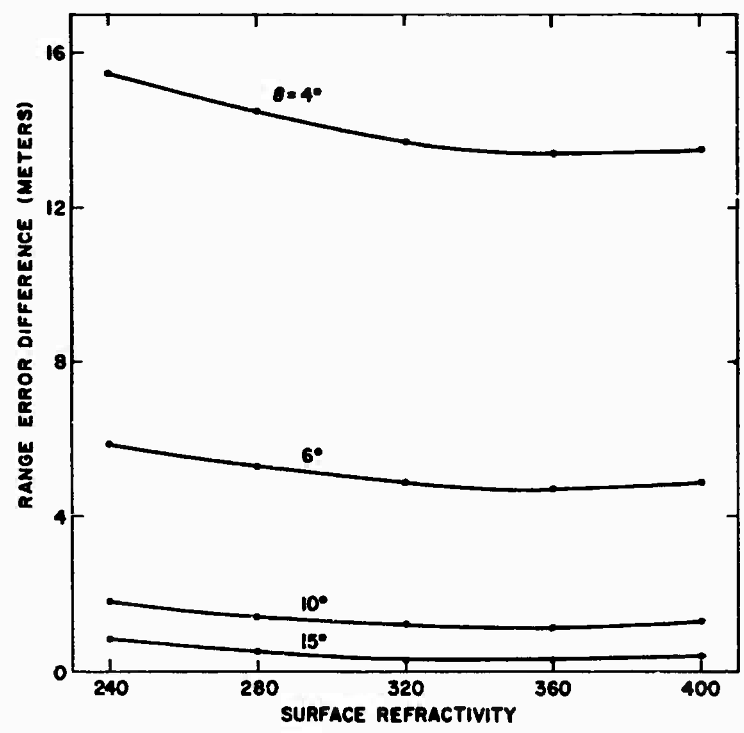

Figure 2. Differences Between Approximate and Accurate Expressions for Range Error as a Function of Surface Refractivity (θ constant)

The next step is to obtain an expression for range error for an aircraft at altitude h through the total atmosphere.

Range error for a path from sea level to an aircraft at altitude h has been computed by Millman (1961, pp.51-52), from whose results it should be possible to estimate the range error for the path between an aircraft and a satellite. For example, if the total range error from sea level to a satellite is ΔR_{eo}, and the range error from sea level to an altitude h is $\Delta R_e'$, then the range error ΔR_{eh} for an aircraft at altitude h to the satellite is approximately $\Delta R_{eh} = \Delta R_{eo} - \Delta R_e'$. From Figure 3, where Millman's (1970) range errors are plotted as a function of altitude for several elevation angles and surface refractivities of N = 280 and N = 360, it can be seen that the height dependence of range error is almost exponential at low altitudes but falls off more rapidly at higher altitudes.

In Appendix B it is shown that the range error from an aircraft at altitude h through the total troposphere to a satellite can be approximated by

$$\Delta R_{eh} = \Delta R_{eo} \exp - \left[(6.07 \times 10^{-5} N_s + 0.0213) h + \frac{0.077}{N_s} - 1.58 \times 10^{-4}) h^2 \right] . \quad (6)$$

Equation (6) yields range errors that agree with Millman's values to within ±0.7 ft. It has now been shown that range errors in excellent agreement with those obtained based on the CRPL Reference Atmosphere 1958 can be computed quite easily from Eq. (6) for elevation angles greater than 5^o. These have been tabulated in Appendix C.

It has been assumed that the surface refractivity is known. The accepted expression (Bean and Dutton, 1966, p.7) for the calculation of refractivity from temperature, pressure, and water vapor pressure is

$$N(h) = 77.6 \frac{P(h)}{T(h)} + 3.73 \times 10^{-5} \frac{e(h)}{T(h)^2} , \quad (7)$$

where

P(h) = pressure in millibars at altitude h,
T(h) = temperature in degrees Kelvin at altitude h,
e(h) = water vapor pressure in millibars at altitude h.

For an aircraft at altitude h, the refractivity is related to the surface refractivity by

$$N(h) = N_s \exp(-h/H) ,$$

where H is a scale height of about 7 km, (Bean and Dutton, 1966, p.15). Therefore $N_s = N(h) \exp(0.043h)$, where h is in thousands of feet.

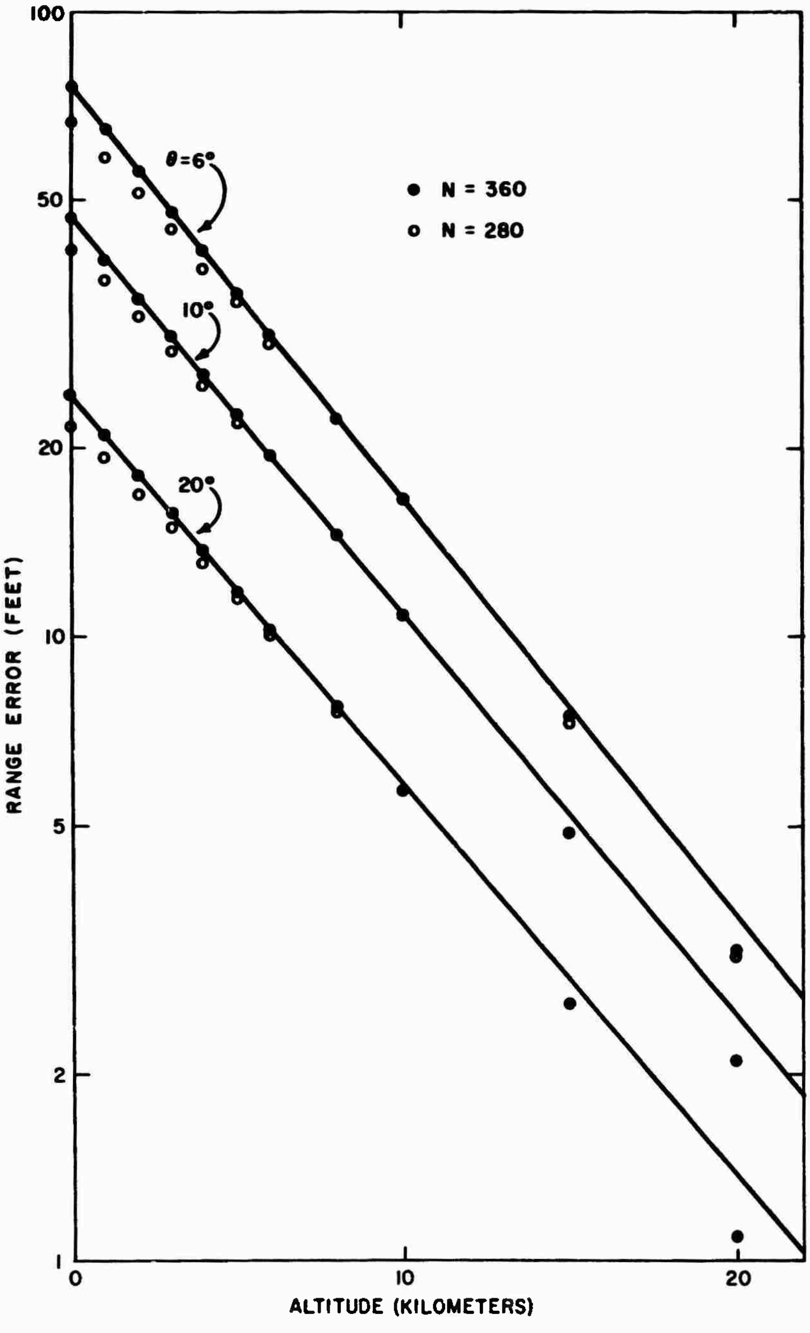

Figure 3. Range Error as a Function of Altitude for Selected Angles and Surface Refractivities

If the temperature and pressure are known, but the water vapor pressure is unknown, it becomes necessary to estimate the refractivity from average values of relative humidity. Bean and Dutton (1966, pp. 398-403) have tabulated refractivity as a function of temperature and relative humidity for pressures of 1000, 850, and 700 millibars, which correspond approximately to altitudes of 0, 5000, and 10000 ft, respectively. From the U.S. Standard Atmosphere Supplements (1966, p. 12) it can be seen that for all northern latitudes—and presumably for all southern latitudes—the average relative humidities at 0, 5000, and 10000 ft are respectively 78, 70, and 50 percent.

In Appendix D the wet term contribution to the refractivity is listed as a function of temperature and pressure. It is shown that it can be approximated by the function

$$N_w(h) = \exp\{0.0824[T(h)-232] - 1.88\times 10^{-4}[T(h)-232]^2 - 4.5h^2[T(h)-203]\times 10^{-5}\} . \tag{8}$$

Total refractivity for the case where the relative humidity is unknown is therefore

$$\begin{aligned} N(h) &= N_d(h) + N_w(h) \\ &= 77.6\,\frac{P(h)}{T(h)} + \exp\{0.0824[T(h)-232] - 1.88\times 10^{-4}[T(h)-232]^2 - 4.5h^2[T(h)-203]\times 10^{-5}\} . \end{aligned} \tag{9}$$

If real-time meteorologic data are not available, there are two choices. The first is to use an average seasonal refractivity (or temperature, pressure, relative humidity) for the location of the aircraft; the second is to use the refractivities in Table 1, which are calculated from the average summer and winter temperature, pressure, and relative humidity as a function of latitude.

Table 1. Average Surface Refractivity

Latitude (degrees)	January	July
0 to 22.5	369	369
22.5 to 37.5	328	382
37.5 to 52.5	307	344
52.5 to 67.5	310	324
67.5 to 82.5	316	315

4. DISCUSSION OF ERRORS

It is extremely difficult to determine the accuracy with which tropospheric range error can be estimated since it is not practicable to measure range error directly. Although range error can be accurately calculated by means of ray-tracing techniques if the refractivity along the path is known, it is not always possible to ascertain atmospheric refractivity since it is variously affected by winds, clouds, and precipitation. In addition, the assumption that the atmosphere is horizontally stratified is not always valid. It is particularly difficult to accurately measure refractivity at the higher altitudes. There is no uniformity of sensors used in radiosondes; although temperature and pressure sensors generally show fair agreement, humidity sensors are erratic (Bean and Dutton, 1966, pp. 23-47). Refractometers are more capable of accurately measuring refractivities, but they are relatively expensive and complex to operate. With these limitations in mind, range errors ΔR_e that had been calculated for the CRPL Standard Sample were examined by Norton (1964), who plotted the standard deviation of $\Delta R_e/R_e$ as a function of elevation angle for ranges of $R_e = 10$, 100, and 1000 km for cases where the surface refractivity is both known and unknown. The curves corresponding to $R_e = 1000$ km were selected for the work of determining the range error to a satellite, and the corresponding standard deviations of ΔR_e plotted in Figure 4.

If it is assumed that the range errors obtained by using the CRPL Reference Atmosphere 1958 have approximately the same standard deviation as those obtained by using the CRPL Standard Sample, then the curves in Figure 4 can be used to estimate the accuracy of the range error corrections. With N_s known, the standard deviations range from 1.3 ft at an elevation angle of 5° to about 0.1 ft at 90°. With N_s unknown, the standard deviations increase to 6.7 ft and 0.7 ft respectively. If the temperature and pressure are known but the humidity is unknown, the standard deviation is presumed to lie somewhere between these two ranges.

Equation (6) has been shown to produce range error corrections that differ by less than 1 ft from the more accurate values of Millman (1970). This error must also be taken into account.

At best, only a qualitative estimate of the accuracy of range error corrections can be presented. The largest uncertainty exists in cases of low elevation angles. If N_s is known, typical range error corrections accurate to 2 or 3 ft can be expected; in rare cases (3σ) they may approach 10 ft. If N_s is unknown, the accuracy of the range error correction decreases significantly, with typical errors on the order of 8 ft and atypical errors approaching 20 ft. These accuracies are consistent with those estimated by Freeman (1964) and Evans (1971).

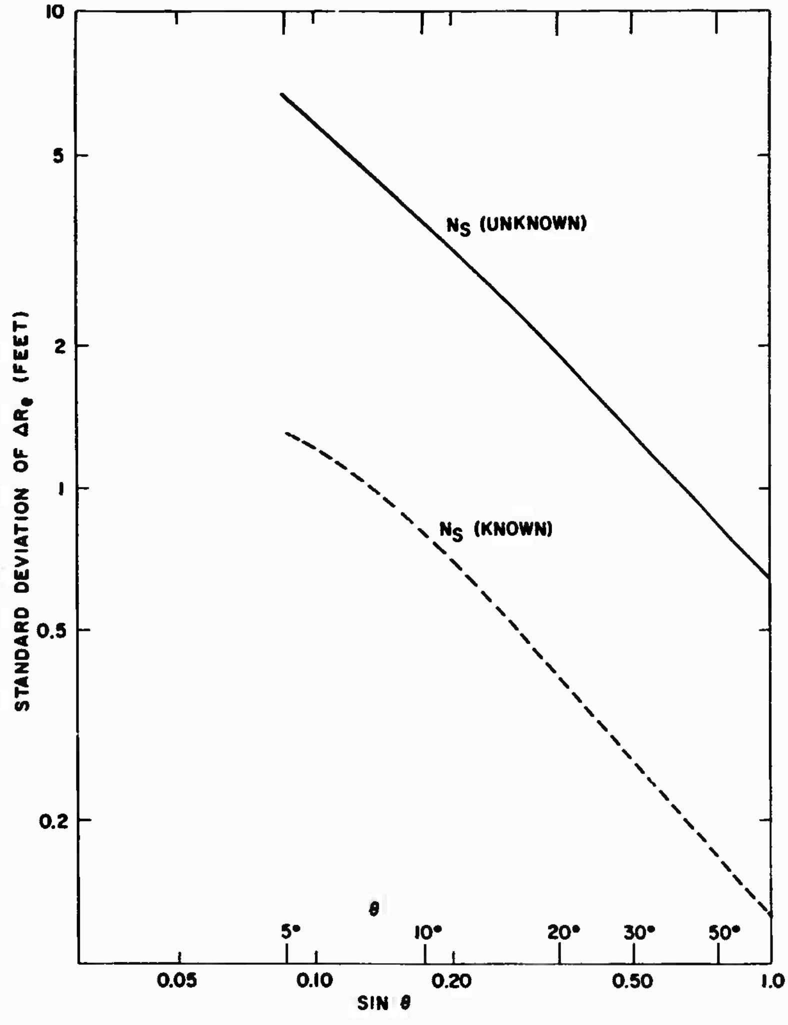

Figure 4. Standard Deviation of Range Error as a Function of Elevation Angle

5. COMMENTS

Tropospheric refraction produces range errors approaching 100 ft for ground-to-satellite paths at elevation angles of 5^{o}; if these errors go uncorrected they can seriously degrade air traffic control and navigation systems based on time delay measurements.

For calculating range error corrections, the CRPL Reference Atmosphere 1958 was selected as the most suitable of the several models of refractivity profiles that were reviewed. The complexity of this model generally requires

numerical methods and a large-capacity digital computer for accurately calculating range error, particularly for low elevation angles. The simple regression lines that have been derived yield range error corrections within about 1 percent of these more accurate values for elevation angles 5° and above, allowing real-time computation of range error with a small computer.

The true accuracy of the range error correction remains unknown, for reasons stated in Sec. 4. On the basis of range error statistics, however, it is estimated that when the surface refractivity is known, the calculated values are probably close to the true values 90 percent of the time; and that when the surface refractivity is unknown, then errors on the order of 10 percent of the true value can be expected.

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Appendix A

Correction of Range Error From Sea Level

As seen in Figure 1, the range error difference Δ has an elevation angle dependence that is approximately linear on a log-log scale. Therefore, Δ should have the form:

$$\ln \Delta(\theta, N_s \text{ constant}) = \ln c_1 + c_2 \ln \theta \ ,$$

or,

$$\Delta(\theta, N_s \text{ constant}) = c_1 \theta^{c_2} \ .$$

With N = 240,

$$\text{for } \theta = 6^\circ: \quad \ln 5.9 = \ln c_1 + c_2 \ln 6^\circ \ ;$$

$$\text{for } \theta = 10^\circ: \quad \ln 1.8 = \ln c_1 + c_2 \ln 10^\circ \ .$$

Solving for c_1 and c_2 yields

$$c_1 = 378 \ ;$$

$$c_2 = -2.30 \ .$$

Therefore,

$$\Delta(\theta, N_s = 240) = 378\theta^{-2.30} .$$

In Figure 2 the range error difference was seen to have a refractivity dependence that is slightly quadratic, with a minimum at about 360. Therefore, $\Delta(N_s, \theta$ constant) should have the form

$$\Delta(N_s, \theta \text{ constant}) = c_3(360 - N_s)^2 + c_4 .$$

With $\theta = 6^o$,

$$\text{for } N_s = 280: \quad 5.3 = c_3(360-280)^2 + c_4 ;$$

$$\text{for } N_s = 360: \quad 4.7 = c_4 .$$

Therefore,

$$c_3 = 9.38 \times 10^{-5} ,$$

$$\Delta(N_s, \theta=6^o) = 9.38 \times 10^{-5} (N_s - 360)^2 + 4.7 .$$

To combine these results into a single expression, noting that Δ has a stronger θ-dependence than N-dependence, let it be required that

$$c_5[\Delta(N_s, \theta=6^o)] = 378, \quad \text{for } N = 240,$$

$$c_5[9.38 \times 10^{-5} (240-360)^2 + 4.7] = 378,$$

$$c_5 = 62.5 .$$

Then

$$\Delta(\theta, N_s) = [0.00586 (N_s - 360)^2 + 294]\, \theta^{-2.30} . \qquad [(4)]$$

Appendix B

Correction of Range Error From an Aircraft

Range errors plotted versus altitude on semilog paper (Figure 3) are seen to be almost linear, which indicates an exponential altitude dependence. To compensate for the nonlinearity at the higher altitudes, an additional term is added. The expression for range error as a function of height is then

$$\ln \Delta R_{eh} = \ln \Delta R_{eo} + c_1 h + c_2 h^2 .$$

The term c_1 is determined from values of ΔR_{eh} at h = 0 and h = 2 km; c_2 is determined from values of ΔR_{eh} at h = 0 and h = 20 km. Thus,

$$c_1 = \frac{\ln \Delta R_{e2} - \ln \Delta R_{eo}}{2} ,$$

$$c_2 = \frac{\ln \Delta R_{e20} - \ln \Delta R_{eo} - 20\, k_1}{400} .$$

These have been computed for N_s = 280 and N_s = 360 at five elevation angles, as follows:

	$N_s=280$		$N_s=360$	
	c_1	c_2	c_1	c_2
6	-0.130	-0.0012	-0.143	-0.0007
10	-0.126	-0.0012	-0.143	-0.0006
15	-0.123	-0.0014	-0.141	-0.0006
20	-0.122	-0.0014	-0.140	-0.0004
30	-0.121	-0.0012	-0.139	-0.0009

On the basis of the above results, the constants

for $N_s = 280$ are taken as: $c_1 = -0.126$, $c_2 = -0.0013$;

for $N_s = 360$ are taken as: $c_1 = -0.142$, $c_2 = -0.0006$.

Since these constants vary slightly with N_s, a linear dependence is assumed, as follows:

$$280\, a_1 + a_o = -0.126$$

$$360\, a_1 + a_o = -0.142$$

$$a_o = -0.07, \quad a_1 = -0.0002$$

$$\frac{b_1}{280} + b_o = -0.0013$$

$$\frac{b_1}{360} + b_o = -0.0006$$

$$b_o = 0.0017, \quad b_1 = -0.83$$

$$c_1 = -0.07 - 0.0002\, N_s$$

$$c_2 = -\frac{0.83}{N_s} + 0.0017$$

$$\Delta R_{eh} = \Delta R_{eo} \exp - \left[(0.0002 N_s + 0.07)h + \left(\frac{0.83}{N_s} + -0.0017\right)h^2\right],$$

where h is in kilometers. Converting h into thousands of feet yields

$$\Delta R_{eh} = \Delta R_{eo} \exp - \left[(6.07 \times 10^{-5} N_s + 0.0213)h + \left(\frac{0.077}{N_s} - 1.58 \times 10^{-4}\right)h^2\right]. \quad [(6)]$$

Appendix C

Table of Range Errors

RANGE ERRORS (feet)
for h = sea level

N units →	240.0	260.0	280.0	300.0	32	340.0	360.0	380.0	400.0
THETA									
4.0	86.52	90.37	94.03	97.49	100.76	103.84	106.72	109.42	111.91
5.0	72.40	75.26	78.02	80.65	83.17	85.58	87.86	90.04	92.39
6.0	62.01	64.29	66.49	68.62	70.67	72.64	74.54	76.36	78.11
7.0	54.14	56.03	57.87	59.65	61.38	63.06	64.68	66.25	67.76
8.0	48.02	49.63	51.20	52.74	54.23	55.69	57.10	58.48	59.82
9.0	43.12	44.53	45.90	47.25	48.57	49.86	51.12	52.34	53.54
10.0	39.12	40.37	41.60	42.80	43.98	45.13	46.27	47.37	48.46
12.0	33.01	34.03	35.04	36.03	37.00	37.96	38.90	39.83	40.74
14.0	28.57	29.43	30.28	31.13	31.96	32.78	33.59	34.38	35.17
16.0	25.20	25.95	26.69	27.42	28.15	28.86	29.57	30.27	30.97
18.0	22.56	23.22	23.88	24.53	25.17	25.81	26.44	27.07	27.69
20.0	20.44	21.03	21.62	22.21	22.79	23.37	23.94	24.50	25.06
25.0	16.62	17.10	17.57	18.04	18.51	18.97	19.43	19.89	20.35
30.0	14.09	14.49	14.89	15.28	15.68	16.07	16.46	16.85	17.23
40.0	11.00	11.31	11.61	11.92	12.23	12.53	12.83	13.13	13.43
50.0	9.25	9.50	9.76	10.02	10.27	10.53	10.78	11.03	11.29
60.0	8.19	8.42	8.64	8.87	9.09	9.32	9.54	9.77	9.99
70.0	7.55	7.76	7.97	8.18	8.39	8.59	8.80	9.01	9.21
80.0	7.21	7.41	7.61	7.81	8.00	8.20	8.40	8.60	8.79
90.0	7.11	7.30	7.50	7.69	7.89	8.08	8.27	8.47	8.66

RANGE ERRORS (feet)
for h = 2000 ft

N units →	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	80.48	83.87	87.05	90.05	92.85	95.46	97.88	100.11	102.15
5.0	67.34	69.84	72.23	74.49	76.64	78.67	80.58	82.38	84.06
6.0	57.68	59.66	61.56	63.38	65.12	66.78	68.36	69.87	71.29
7.0	50.36	52.00	53.58	55.10	56.56	57.97	59.32	60.61	61.85
8.0	44.66	46.06	47.40	48.71	49.97	51.19	52.37	53.51	54.60
9.0	40.11	41.32	42.50	43.64	44.76	45.83	46.88	47.89	48.87
10.0	36.39	37.47	38.51	39.53	40.53	41.49	42.43	43.34	44.23
12.0	30.71	31.58	32.44	33.28	34.10	34.90	35.68	36.44	37.19
14.0	26.57	27.31	28.04	28.75	29.45	30.13	30.80	31.46	32.10
16.0	23.44	24.08	24.71	25.33	25.94	26.53	27.12	27.70	28.27
18.0	20.98	21.55	22.11	22.66	23.20	23.73	24.25	24.77	25.27
20.0	19.01	19.52	20.02	20.51	21.00	21.48	21.95	22.42	22.88
25.0	15.46	15.87	16.27	16.66	17.05	17.44	17.82	18.20	18.57
30.0	13.11	13.45	13.78	14.12	14.45	14.77	15.09	15.41	15.73
40.0	10.23	10.49	10.75	11.01	11.26	11.52	11.77	12.02	12.26
50.0	8.60	8.82	9.04	9.25	9.47	9.68	9.89	10.10	10.30
60.0	7.62	7.81	8.00	8.19	8.38	8.57	8.75	8.94	9.12
70.0	7.03	7.20	7.38	7.55	7.73	7.90	8.07	8.24	8.41
80.0	6.71	6.88	7.04	7.21	7.38	7.54	7.70	7.87	8.03
90.0	6.61	6.77	6.94	7.10	7.27	7.43	7.59	7.75	7.91

RANGE ERRORS (feet)
for h = 4000 ft

N units →	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	74.76	77.74	80.52	83.11	85.50	87.71	89.72	91.56	93.21
5.0	62.56	64.74	66.81	68.75	70.57	72.28	73.87	75.34	76.70
6.0	53.58	55.30	56.94	58.49	59.96	61.36	62.67	63.90	65.05
7.0	46.79	48.20	49.56	50.85	52.08	53.26	54.38	55.43	56.44
8.0	41.49	42.69	43.85	44.96	46.02	47.04	48.01	48.94	49.82
9.0	37.26	38.30	39.31	40.28	41.21	42.11	42.97	43.80	44.59
10.0	33.81	34.73	35.62	36.49	37.32	38.12	38.90	39.64	40.36
12.0	28.53	29.28	30.00	30.71	31.40	32.06	32.71	33.33	33.93
14.0	24.68	25.32	25.93	26.53	27.12	27.68	28.24	28.77	29.29
16.0	21.77	22.32	22.85	23.38	23.88	24.38	24.86	25.33	25.79
18.0	19.49	19.98	20.45	20.91	21.36	21.80	22.23	22.65	23.06
20.0	17.66	18.09	18.52	18.93	19.34	19.73	20.12	20.50	20.87
25.0	14.36	14.71	15.05	15.38	15.70	16.02	16.34	16.64	16.95
30.0	12.18	12.46	12.75	13.03	13.30	13.57	13.84	14.10	14.35
40.0	9.50	9.73	9.95	10.16	10.37	10.58	10.79	10.99	11.19
50.0	7.99	8.18	8.36	8.54	8.72	8.89	9.06	9.23	9.40
60.0	7.08	7.24	7.40	7.56	7.72	7.87	8.02	8.17	8.32
70.0	6.53	6.68	6.82	6.97	7.12	7.26	7.40	7.54	7.67
80.0	6.23	6.37	6.52	6.65	6.79	6.93	7.06	7.19	7.32
90.0	6.14	6.28	6.42	6.56	6.69	6.82	6.96	7.09	7.21

RANGE ERRORS (feet)
for h = 6000 ft

N units →	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	69.36	71.98	74.41	76.64	78.68	80.54	82.21	83.71	85.03
5.0	58.04	59.95	61.74	63.40	64.95	66.37	67.68	68.88	69.97
6.0	49.71	51.21	52.62	53.94	55.18	56.34	57.42	58.42	59.34
7.0	43.40	44.63	45.80	46.89	47.93	48.91	49.82	50.68	51.48
8.0	38.49	39.53	40.52	41.46	42.35	43.19	43.99	44.74	45.45
9.0	34.57	35.47	36.33	37.15	37.93	38.67	39.38	40.04	40.68
10.0	31.36	32.16	32.92	33.65	34.34	35.01	35.64	36.24	36.82
12.0	26.46	27.11	27.73	28.32	28.89	29.44	29.97	30.47	30.95
14.0	22.90	23.44	23.96	24.47	24.95	25.42	25.87	26.30	26.72
16.0	20.20	20.67	21.12	21.56	21.98	22.39	22.78	23.16	23.53
18.0	18.08	18.50	18.89	19.28	19.66	20.02	20.37	20.71	21.04
20.0	16.38	16.75	17.11	17.46	17.80	18.12	18.44	18.74	19.04
25.0	13.32	13.62	13.90	14.18	14.45	14.71	14.97	15.22	15.46
30.0	11.30	11.54	11.78	12.01	12.24	12.46	12.68	12.89	13.09
40.0	8.82	9.01	9.19	9.37	9.55	9.72	9.88	10.05	10.21
50.0	7.41	7.57	7.72	7.87	8.02	8.16	8.30	8.44	8.57
60.0	6.56	6.70	6.84	6.97	7.10	7.23	7.35	7.47	7.59
70.0	6.05	6.18	6.31	6.43	6.55	6.66	6.78	6.89	7.00
80.0	5.78	5.90	6.02	6.14	6.25	6.36	6.47	6.58	6.68
90.0	5.70	5.81	5.93	6.05	6.16	6.27	6.37	6.48	6.58

RANGE ERRORS (feet)
for h = 8000 ft

N units →	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	64.26	66.58	68.70	70.62	72.36	73.92	75.30	76.50	77.54
5.0	53.77	55.45	57.00	58.42	59.73	60.92	61.99	62.95	63.81
6.0	46.06	47.36	48.58	49.71	50.75	51.71	52.59	53.39	54.12
7.0	40.22	41.28	42.28	43.21	44.08	44.89	45.63	46.32	46.95
8.0	35.66	36.56	37.41	38.20	38.95	39.64	40.29	40.89	41.45
9.0	32.03	32.80	33.54	34.23	34.88	35.49	36.06	36.60	37.10
10.0	29.06	29.74	30.39	31.00	31.58	32.13	32.64	33.12	33.58
12.0	24.52	25.07	25.60	26.10	26.57	27.02	27.45	27.85	28.23
14.0	21.22	21.68	22.12	22.55	22.95	23.33	23.69	24.04	24.37
16.0	18.71	19.12	19.50	19.86	20.21	20.55	20.86	21.17	21.46
18.0	16.75	17.11	17.44	17.77	18.08	18.37	18.66	18.93	19.18
20.0	15.18	15.50	15.80	16.09	16.36	16.63	16.89	17.13	17.37
25.0	12.34	12.60	12.84	13.07	13.29	13.50	13.71	13.91	14.10
30.0	10.47	10.67	10.88	11.07	11.26	11.44	11.61	11.78	11.94
40.0	8.17	8.33	8.48	8.63	8.78	8.92	9.05	9.18	9.31
50.0	6.87	7.00	7.13	7.25	7.38	7.49	7.60	7.71	7.82
60.0	6.08	6.20	6.31	6.42	6.53	6.63	6.73	6.83	6.92
70.0	5.61	5.72	5.82	5.92	6.02	6.12	6.21	6.30	6.38
80.0	5.36	5.46	5.56	5.65	5.75	5.84	5.93	6.01	6.09
90.0	5.28	5.38	5.48	5.57	5.66	5.75	5.84	5.92	6.00

RANGE ERRORS (feet)
for h = 10000 ft

N units→	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	59.47	61.51	63.37	65.03	66.50	67.80	68.93	69.90	70.70
5.0	49.76	51.23	52.57	53.79	54.89	55.88	56.75	57.52	58.18
6.0	42.62	43.76	44.81	45.77	46.64	47.43	48.14	48.78	49.34
7.0	37.21	38.14	39.00	39.79	40.51	41.17	41.77	42.32	42.81
8.0	33.00	33.78	34.50	35.17	35.79	36.36	36.88	37.36	37.79
9.0	29.63	30.31	30.93	31.52	32.05	32.55	33.01	33.44	33.82
10.0	26.89	27.48	28.03	28.55	29.03	29.47	29.88	30.26	30.61
12.0	22.69	23.16	23.61	24.03	24.42	24.78	25.13	25.44	25.74
14.0	19.63	20.03	20.41	20.76	21.09	21.40	21.69	21.96	22.22
16.0	17.32	17.66	17.98	18.29	18.58	18.85	19.10	19.34	19.56
18.0	15.50	15.80	16.09	16.36	16.61	16.85	17.08	17.29	17.49
20.0	14.05	14.32	14.57	14.81	15.04	15.26	15.46	15.65	15.83
25.0	11.42	11.64	11.84	12.03	12.21	12.39	12.55	12.71	12.85
30.0	9.68	9.86	10.03	10.19	10.35	10.49	10.63	10.76	10.88
40.0	7.56	7.70	7.83	7.95	8.07	8.18	8.29	8.39	8.49
50.0	6.35	6.47	6.58	6.68	6.78	6.87	6.96	7.05	7.13
60.0	5.63	5.73	5.82	5.91	6.00	6.08	6.16	6.24	6.31
70.0	5.19	5.28	5.37	5.45	5.53	5.61	5.68	5.75	5.82
80.0	4.96	5.04	5.13	5.21	5.28	5.35	5.42	5.49	5.55
90.0	4.88	4.97	5.05	5.13	5.20	5.27	5.34	5.41	5.47

RANGE ERRORS (feet)
for h = 15000 ft

N units→	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	48.70	50.23	51.56	52.71	53.69	54.51	55.17	55.68	56.05
5.0	40.75	41.83	42.78	43.61	44.32	44.92	45.42	45.82	46.12
6.0	34.90	35.73	36.46	37.10	37.66	38.13	38.53	38.86	39.12
7.0	30.48	31.14	31.73	32.25	32.71	33.10	33.43	33.71	33.94
8.0	27.03	27.58	28.08	28.51	28.90	29.23	29.52	29.76	29.96
9.0	24.27	24.75	25.17	25.55	25.88	26.17	26.42	26.63	26.81
10.0	22.02	22.44	22.81	23.14	23.43	23.69	23.91	24.11	24.27
12.0	18.58	18.91	19.21	19.48	19.72	19.92	20.11	20.27	20.40
14.0	16.08	16.36	16.61	16.83	17.03	17.20	17.36	17.50	17.61
16.0	14.18	14.42	14.63	14.83	15.00	15.15	15.28	15.40	15.51
18.0	12.70	12.90	13.09	13.26	13.41	13.55	13.67	13.77	13.87
20.0	11.50	11.69	11.86	12.01	12.14	12.26	12.37	12.47	12.55
25.0	9.35	9.50	9.63	9.75	9.86	9.96	10.04	10.12	10.19
30.0	7.93	8.05	8.16	8.26	8.35	8.43	8.51	8.57	8.63
40.0	6.19	6.28	6.37	6.44	6.51	6.57	6.63	6.68	6.73
50.0	5.20	5.28	5.35	5.41	5.47	5.52	5.57	5.61	5.65
60.0	4.61	4.68	4.74	4.79	4.84	4.89	4.93	4.97	5.00
70.0	4.25	4.31	4.37	4.42	4.47	4.51	4.55	4.58	4.61
80.0	4.06	4.12	4.17	4.22	4.26	4.30	4.34	4.37	4.40
90.0	4.00	4.06	4.11	4.16	4.20	4.24	4.28	4.31	4.34

RANGE ERRORS (feet)
for h = 20000 ft

N units →	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	39.56	40.73	41.71	42.52	43.17	43.67	44.03	44.25	44.36
5.0	33.10	33.92	34.61	35.18	35.63	35.99	36.25	36.41	36.50
6.0	28.35	28.97	29.50	29.93	30.28	30.55	30.75	30.88	30.96
7.0	24.76	25.25	25.67	26.02	26.30	26.52	26.68	26.79	26.86
8.0	21.95	22.37	22.71	23.00	23.23	23.42	23.56	23.65	23.71
9.0	19.71	20.07	20.36	20.61	20.81	20.97	21.08	21.17	21.22
10.0	17.89	18.19	18.45	18.67	18.84	18.98	19.08	19.16	19.20
12.0	15.09	15.34	15.54	15.71	15.85	15.96	16.05	16.11	16.15
14.0	13.06	13.26	13.43	13.57	13.69	13.78	13.85	13.90	13.94
16.0	11.52	11.69	11.84	11.96	12.06	12.14	12.20	12.24	12.27
18.0	10.31	10.46	10.59	10.70	10.78	10.85	10.91	10.95	10.97
20.0	9.34	9.48	9.59	9.68	9.76	9.82	9.87	9.91	9.93
25.0	7.60	7.70	7.79	7.87	7.93	7.98	8.01	8.04	8.06
30.0	6.44	6.53	6.60	6.66	6.71	6.76	6.79	6.81	6.83
40.0	5.03	5.09	5.15	5.20	5.24	5.27	5.29	5.31	5.32
50.0	4.23	4.28	4.33	4.37	4.40	4.42	4.44	4.46	4.47
60.0	3.74	3.79	3.83	3.87	3.89	3.92	3.93	3.95	3.96
70.0	3.45	3.50	3.53	3.56	3.59	3.61	3.63	3.64	3.65
80.0	3.30	3.34	3.37	3.40	3.43	3.45	3.46	3.47	3.48
90.0	3.25	3.29	3.32	3.35	3.38	3.40	3.41	3.42	3.43

RANGE ERRORS (feet)
for h = 25000 ft

N units →	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	31.87	32.80	33.55	34.13	34.57	34.87	35.04	35.09	35.04
5.0	26.67	27.32	27.83	28.24	28.53	28.73	28.85	28.88	28.84
6.0	22.84	23.33	23.72	24.02	24.24	24.39	24.47	24.49	24.46
7.0	19.95	20.34	20.65	20.88	21.06	21.17	21.23	21.25	21.22
8.0	17.69	18.01	18.27	18.46	18.60	18.70	18.75	18.76	18.73
9.0	15.88	16.16	16.38	16.54	16.66	16.74	16.78	16.79	16.76
10.0	14.41	14.65	14.84	14.98	15.09	15.15	15.19	15.19	15.17
12.0	12.16	12.35	12.50	12.61	12.69	12.74	12.77	12.77	12.76
14.0	10.52	10.68	10.80	10.90	10.96	11.00	11.02	11.03	11.01
16.0	9.28	9.42	9.52	9.60	9.65	9.69	9.71	9.71	9.69
18.0	8.31	8.43	8.52	8.59	8.63	8.66	8.68	8.68	8.67
20.0	7.53	7.63	7.71	7.77	7.82	7.84	7.86	7.86	7.85
25.0	6.12	6.20	6.27	6.31	6.35	6.37	6.38	6.38	6.37
30.0	5.19	5.26	5.31	5.35	5.38	5.39	5.40	5.40	5.39
40.0	4.05	4.10	4.14	4.17	4.19	4.20	4.21	4.21	4.20
50.0	3.40	3.45	3.48	3.50	3.52	3.53	3.54	3.54	3.53
60.0	3.01	3.05	3.08	3.10	3.12	3.13	3.13	3.13	3.13
70.0	2.78	2.81	2.84	2.86	2.87	2.88	2.89	2.89	2.88
80.0	2.65	2.69	2.71	2.73	2.74	2.75	2.75	2.75	2.75
90.0	2.61	2.65	2.67	2.69	2.70	2.71	2.71	2.71	2.71

RANGE ERRORS (feet)
for h = 30000 ft

N units→ THETA	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
4.0	25.47	26.23	26.82	27.26	27.57	27.74	27.81	27.77	27.64
5.0	21.31	21.85	22.25	22.55	22.75	22.86	22.89	22.85	22.74
6.0	18.26	18.66	18.97	19.19	19.33	19.41	19.42	19.38	19.29
7.0	15.94	16.26	16.51	16.68	16.79	16.84	16.85	16.81	16.73
8.0	14.14	14.40	14.60	14.75	14.83	14.88	14.88	14.84	14.77
9.0	12.69	12.92	13.09	13.21	13.29	13.32	13.32	13.28	13.22
10.0	11.52	11.72	11.87	11.97	12.03	12.06	12.05	12.02	11.96
12.0	9.72	9.88	9.99	10.07	10.12	10.14	10.13	10.11	10.06
14.0	8.41	8.54	8.64	8.70	8.74	8.75	8.75	8.72	8.68
16.0	7.42	7.53	7.61	7.67	7.70	7.71	7.70	7.68	7.64
18.0	6.64	6.74	6.81	6.86	6.88	6.89	6.89	6.87	6.83
20.0	6.02	6.10	6.17	6.21	6.23	6.24	6.23	6.22	6.19
25.0	4.89	4.96	5.01	5.04	5.06	5.07	5.06	5.05	5.02
30.0	4.15	4.20	4.24	4.27	4.29	4.29	4.28	4.27	4.25
40.0	3.24	3.28	3.31	3.33	3.34	3.34	3.34	3.33	3.31
50.0	2.72	2.76	2.78	2.80	2.81	2.81	2.81	2.80	2.78
60.0	2.41	2.44	2.46	2.48	2.48	2.49	2.48	2.48	2.46
70.0	2.22	2.25	2.27	2.28	2.29	2.29	2.29	2.28	2.27
80.0	2.12	2.15	2.17	2.18	2.19	2.19	2.19	2.18	2.17
90.0	2.09	2.12	2.14	2.15	2.15	2.16	2.15	2.15	2.14

RANGE ERRORS (feet)
for h = 40000 ft

N units→	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	15.88	16.43	16.85	17.14	17.31	17.38	17.37	17.27	17.10
5.0	13.28	13.69	13.98	14.18	14.29	14.33	14.30	14.21	14.07
6.0	11.38	11.69	11.91	12.06	12.14	12.16	12.13	12.05	11.93
7.0	9.93	10.19	10.37	10.48	10.54	10.55	10.52	10.45	10.35
8.0	8.81	9.02	9.17	9.27	9.32	9.32	9.29	9.23	9.14
9.0	7.91	8.10	8.22	8.30	8.34	8.34	8.32	8.26	8.18
10.0	7.18	7.34	7.45	7.52	7.55	7.55	7.53	7.47	7.40
12.0	6.06	6.19	6.28	6.33	6.35	6.35	6.33	6.28	6.22
14.0	5.24	5.35	5.42	5.47	5.49	5.48	5.46	5.42	5.37
16.0	4.62	4.72	4.78	4.82	4.83	4.83	4.81	4.77	4.73
18.0	4.14	4.22	4.28	4.31	4.32	4.32	4.30	4.27	4.23
20.0	3.75	3.82	3.87	3.90	3.91	3.91	3.89	3.86	3.83
25.0	3.05	3.11	3.15	3.17	3.18	3.17	3.16	3.14	3.11
30.0	2.58	2.63	2.66	2.68	2.69	2.69	2.67	2.66	2.63
40.0	2.01	2.05	2.08	2.09	2.10	2.09	2.08	2.07	2.05
50.0	1.69	1.72	1.75	1.76	1.76	1.76	1.75	1.74	1.72
60.0	1.50	1.53	1.54	1.56	1.56	1.56	1.55	1.54	1.52
70.0	1.38	1.41	1.42	1.43	1.44	1.43	1.43	1.42	1.40
80.0	1.32	1.34	1.36	1.37	1.37	1.37	1.36	1.35	1.34
90.0	1.30	1.32	1.34	1.35	1.35	1.35	1.34	1.33	1.32

RANGE ERRORS (feet)
for h = 50000 ft

N units →	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	9.58	10.01	10.34	10.56	10.69	10.74	10.72	10.64	10.51
5.0	8.01	8.34	8.58	8.74	8.83	8.85	8.83	8.76	8.65
6.0	6.86	7.12	7.31	7.43	7.50	7.51	7.49	7.43	7.33
7.0	5.99	6.21	6.36	6.46	6.51	6.52	6.50	6.44	6.36
8.0	5.31	5.50	5.63	5.71	5.75	5.76	5.74	5.69	5.61
9.0	4.77	4.93	5.04	5.12	5.15	5.16	5.13	5.09	5.02
10.0	4.33	4.47	4.57	4.63	4.66	4.67	4.65	4.61	4.55
12.0	3.65	3.77	3.85	3.90	3.92	3.92	3.91	3.87	3.82
14.0	3.16	3.26	3.33	3.37	3.39	3.39	3.37	3.34	3.30
16.0	2.79	2.87	2.93	2.97	2.98	2.98	2.97	2.94	2.90
18.0	2.49	2.57	2.62	2.65	2.67	2.67	2.65	2.63	2.60
20.0	2.26	2.33	2.37	2.40	2.41	2.41	2.40	2.38	2.35
25.0	1.84	1.89	1.93	1.95	1.96	1.96	1.95	1.93	1.91
30.0	1.56	1.60	1.63	1.65	1.66	1.66	1.65	1.63	1.61
40.0	1.21	1.25	1.27	1.29	1.29	1.29	1.29	1.27	1.26
50.0	1.02	1.05	1.07	1.08	1.09	1.09	1.08	1.07	1.06
60.0	0.90	0.93	0.95	0.96	0.96	0.96	0.95	0.95	0.93
70.0	0.83	0.86	0.87	0.88	0.89	0.89	0.88	0.87	0.86
80.0	0.79	0.82	0.83	0.84	0.85	0.84	0.84	0.83	0.82
90.0	0.78	0.80	0.82	0.83	0.83	0.83	0.83	0.82	0.81

RANGE ERRORS (feet)
for h = 60000 ft

N units →	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	5.59	5.94	6.20	6.38	6.50	6.55	6.55	6.50	6.41
5.0	4.68	4.94	5.14	5.28	5.36	5.40	5.39	5.35	5.27
6.0	4.01	4.22	4.38	4.49	4.55	4.58	4.57	4.53	4.47
7.0	3.50	3.68	3.81	3.90	3.96	3.98	3.97	3.93	3.88
8.0	3.10	3.26	3.37	3.45	3.49	3.51	3.50	3.47	3.42
9.0	2.78	2.92	3.02	3.09	3.13	3.14	3.13	3.11	3.06
10.0	2.53	2.65	2.74	2.80	2.83	2.84	2.84	2.81	2.77
12.0	2.13	2.23	2.31	2.35	2.38	2.39	2.38	2.36	2.33
14.0	1.84	1.93	1.99	2.03	2.06	2.06	2.06	2.04	2.01
16.0	1.63	1.70	1.76	1.79	1.81	1.82	1.81	1.80	1.77
18.0	1.45	1.52	1.57	1.60	1.62	1.62	1.62	1.60	1.58
20.0	1.32	1.38	1.42	1.45	1.47	1.47	1.47	1.45	1.43
25.0	1.07	1.12	1.15	1.18	1.19	1.19	1.19	1.18	1.16
30.0	0.91	0.95	0.98	1.00	1.01	1.01	1.01	1.00	0.98
40.0	0.71	0.74	0.76	0.78	0.78	0.79	0.78	0.78	0.77
50.0	0.59	0.62	0.64	0.65	0.66	0.66	0.66	0.65	0.64
60.0	0.53	0.55	0.57	0.58	0.58	0.58	0.58	0.58	0.57
70.0	0.48	0.51	0.52	0.53	0.54	0.54	0.54	0.53	0.52
80.0	0.46	0.48	0.50	0.51	0.51	0.51	0.51	0.51	0.50
90.0	0.45	0.48	0.49	0.50	0.50	0.51	0.50	0.50	0.49

RANGE ERRORS (feet)
for h = 80000 ft

N units →	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	1.73	1.92	2.07	2.19	2.28	2.33	2.36	2.36	2.34
5.0	1.44	1.60	1.72	1.81	1.88	1.92	1.94	1.94	1.92
6.0	1.24	1.36	1.46	1.54	1.60	1.63	1.65	1.64	1.63
7.0	1.08	1.19	1.27	1.34	1.39	1.42	1.43	1.43	1.41
8.0	0.96	1.05	1.13	1.18	1.22	1.25	1.26	1.26	1.25
9.0	0.86	0.94	1.01	1.06	1.10	1.12	1.13	1.13	1.12
10.0	0.78	0.85	0.91	0.96	0.99	1.01	1.02	1.02	1.01
12.0	0.66	0.72	0.77	0.81	0.83	0.85	0.86	0.86	0.85
14.0	0.57	0.62	0.66	0.70	0.72	0.73	0.74	0.74	0.73
16.0	0.50	0.55	0.58	0.61	0.63	0.65	0.65	0.65	0.64
18.0	0.45	0.49	0.52	0.55	0.57	0.58	0.58	0.58	0.57
20.0	0.40	0.44	0.47	0.50	0.51	0.52	0.53	0.52	0.52
25.0	0.33	0.36	0.38	0.40	0.41	0.42	0.43	0.42	0.42
30.0	0.28	0.30	0.32	0.34	0.35	0.36	0.36	0.36	0.36
40.0	0.22	0.24	0.25	0.26	0.27	0.28	0.28	0.28	0.28
50.0	0.18	0.20	0.21	0.22	0.23	0.23	0.23	0.23	0.23
60.0	0.16	0.17	0.19	0.20	0.20	0.20	0.21	0.21	0.20
70.0	0.15	0.16	0.17	0.18	0.19	0.19	0.19	0.19	0.19
80.0	0.14	0.15	0.16	0.17	0.18	0.18	0.18	0.18	0.18
90.0	0.14	0.15	0.16	0.17	0.17	0.18	0.18	0.18	0.18

RANGE ERRORS (feet)
for h = 100000 ft

N units →	240.0	260.0	280.0	300.0	320.0	340.0	360.0	380.0	400.0
THETA									
4.0	0.47	0.55	0.63	0.69	0.75	0.79	0.81	0.82	0.83
5.0	0.39	0.46	0.52	0.57	0.62	0.65	0.67	0.68	0.68
6.0	0.33	0.39	0.44	0.49	0.52	0.55	0.56	0.57	0.57
7.0	0.29	0.34	0.39	0.42	0.45	0.47	0.49	0.50	0.50
8.0	0.26	0.30	0.34	0.37	0.40	0.42	0.43	0.44	0.44
9.0	0.23	0.27	0.30	0.33	0.36	0.37	0.39	0.39	0.39
10.0	0.21	0.24	0.28	0.30	0.32	0.34	0.35	0.35	0.35
12.0	0.17	0.20	0.23	0.25	0.27	0.28	0.29	0.30	0.30
14.0	0.15	0.18	0.20	0.22	0.23	0.24	0.25	0.26	0.26
16.0	0.13	0.15	0.17	0.19	0.20	0.21	0.22	0.22	0.22
18.0	0.12	0.14	0.16	0.17	0.18	0.19	0.20	0.20	0.20
20.0	0.11	0.12	0.14	0.15	0.16	0.17	0.18	0.18	0.18
25.0	0.09	0.10	0.11	0.12	0.13	0.14	0.14	0.15	0.15
30.0	0.07	0.08	0.10	0.10	0.11	0.12	0.12	0.12	0.12
40.0	0.05	0.06	0.07	0.08	0.09	0.09	0.09	0.09	0.09
50.0	0.05	0.05	0.06	0.07	0.07	0.08	0.08	0.08	0.08
60.0	0.04	0.05	0.05	0.06	0.06	0.07	0.07	0.07	0.07
70.0	0.04	0.04	0.05	0.05	0.06	0.06	0.06	0.06	0.06
80.0	0.03	0.04	0.05	0.05	0.05	0.06	0.06	0.06	0.06
90.0	0.03	0.04	0.05	0.05	0.05	0.06	0.06	0.06	0.06

Appendix D

Wet Term Contribution to Refractivity (Relative Humidity Unknown)

The wet term contribution to refractivity for average relative humidities of 78, 70, and 50 percent, at pressures of 1000, 850, and 700 mb, respectively, is obtained by subtracting the dry term refractivity (zero percent relative humidity) from the total refractivity (Bean and Dutton, 1966, pp. 398-403).

When the wet term of the surface refractivity (N_{ws}) is plotted against surface temperature on semilog paper (Figure D1) it appears that it can be approximated with a regression line of the form

$$\ln N_{ws} = c_1 (T - c_o) + c_2(T - c_o)^2 .$$

Setting

$$T = 243^{o}K, \ln 2.5 = c_1 (243 - c_o) ;$$

$$T = 253^{o}K, \ln 5.7 = c_1 (253 - c_o) ;$$

we get

$$c_o = 232, \; c_1 = 0.0824 .$$

Then for

$$T = 303^{o}K, \ln 134.6 = 0.0824(303\text{-}232) + c_2(303\text{-}232)^2 ,$$

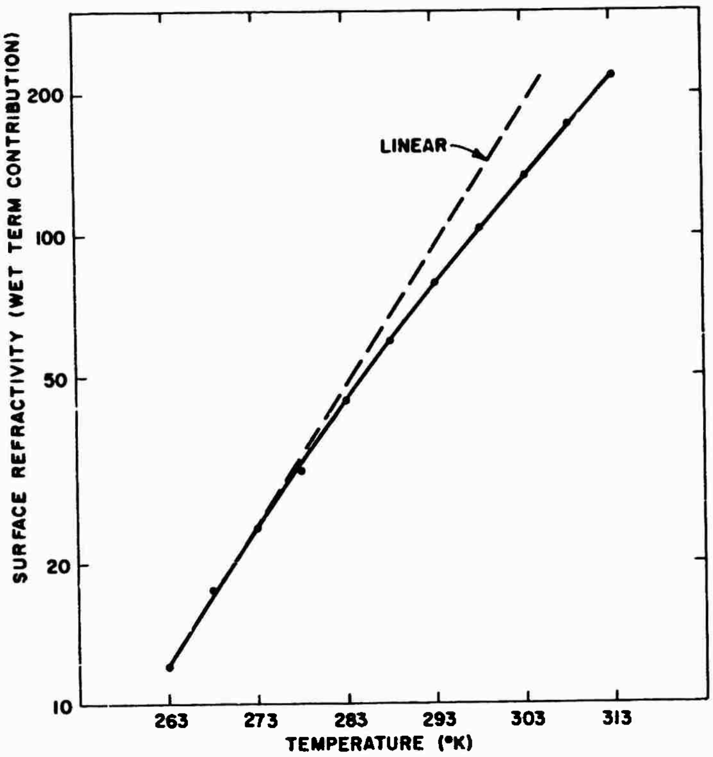

Figure D1. Wet Term Component of Surface Refractivity as a Function of Temperature

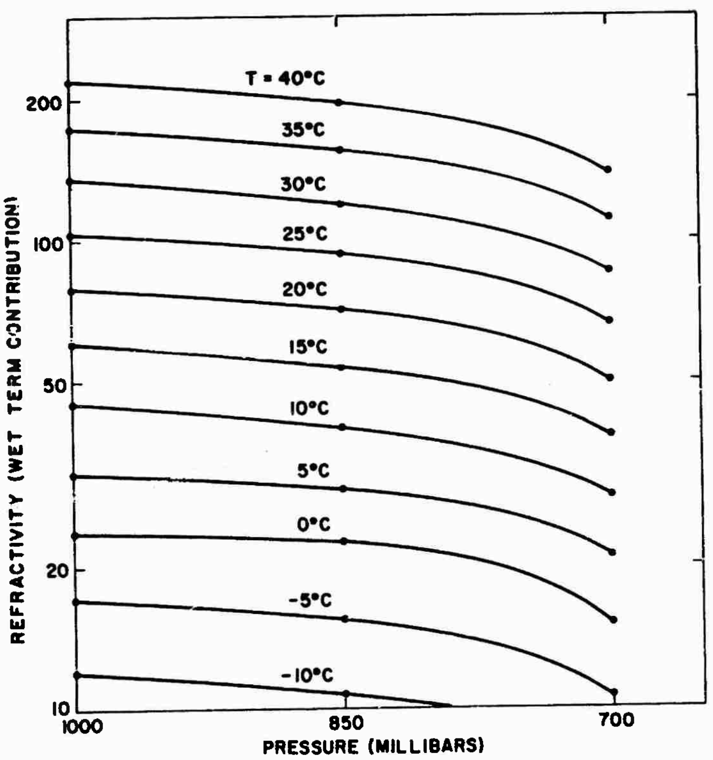

Figure D2. Wet Term Component of Surface Refractivity as a Function of Pressure

we get

$$c_2 = -1.88 \times 10^{-4} .$$

At sea level, therefore,

$$N_{ws} = \exp [0.0824(T-232) - 1.88 \times 10^{-4}(T-232^{2}] .$$

In Figure D2 the wet component of refractivity is plotted as a function of pressure for a series of constant temperatures. It can be seen that this family of curves has a pressure (or height) dependence of the form

$$N_{wh} = N_{ws} \exp (-kh^2) ,$$

where pressures of 1000, 850, and 700 mb are assumed to correspond to altitudes h of approximately 0, 5000, and 10000 ft, respectively, and k is a constant. It can be shown that

$$k = -4.5(T-203) \times 10^{-5} .$$

Thus,

$$N_{wh} = \exp [0.0824(T-232) - 1.88 \times 10^{-4}(T-232)^2 - 4.5\, h^2(T-203) \times 10^{-5}] .$$